Mathematical Modeling. A Chemical Engineer's Perspective

By Rutherford Aris, Academic Press, New York, 1999, xxi + 479 pp., $99.95.

If you enjoy stylishly written prose in which the author boldly states his opinions, then this book is for you. The course of developing an original idea is a messy process, fraught with false starts, wrong turns, blind alleys, mistakes, inspiration, and hard work. When the results are reported in the form of research papers or articles, we rarely describe things in chronological order, and almost never disclose the tortured path taken. Experienced hands will recognize this as they reflect on their own journeys to publication. Although there is a lot to be said in favor of the traditionally cleansed research paper, an unfortunate side effect is that they can be antiseptic, or worse, sterile. It is often difficult to learn deeper lessons on method and style of scientific research by reading research jourals.

In this beautifully written book, Aris uses a selection of his classic papers as a vehicle for teachning his approach to research, and for passing on his many valuable opinions about style. Few people could successfully pull off writing a book like this, but Aris succeeds as only he can.

Aris describes his goals for the book in the preface. It contains: (1) a brief biography that sets the context and acknowledges the people who have significantly influenced his career; (2) a selection of his research papers which provide diverse examples of the methods, applications, and successes of mathematical modeling; and (3) a handful of chapters that "provide a more tutorial exposition of the art of modeling." He describes the book as elementary and idiosyncratic. It may well qualify as idiosyncratic, but in some places it is not as elementary as he proclaims. In this respect, he follows his own course–after publishing his masterpiece, *An Introduction to the Analysis of Chemical Reactors* in 1965, many people suggested it was too difficult for undergraduates and that he should publish a simpler textbook. So in 1969, he published *Elementary Chemical Reactor Analysis*, which was considered to be equally difficult!

Mathematical Modeling is divided into three parts. Part I consists of six chapters on Method and Manner. This is Aris at his best. Part II contains 17 of the author's papers divided into five chapters, and Part III, "Miscellanea," provides a brief autobiography and a complete bibliography through 1999.

The first 100 pages (Part I, pp. 3–103) should be compulsory reading for every serious student of chemical engineering. Aris begins with a dogmatic statement on modeling, "A mathematical model is a representation, in mathematical terms, of certain aspects of a nonmathematical system. The arts and crafts of mathematical modeling are exhibited in the construction of models that not only are consistent in themselves and mirror the behavior of their prototype, but also serve some exterior purpose." In line with his urging later in the book that there is no substitute for practice, he immediately demonstrates his definition of modeling by launching into the development of a model for the continuous stirred tank reactor. He insists that models be put in dimensionless form, so that the influence of parameters becomes more apparent, and the CSTR is one of the simplest devices in chemical engineering that demonstrates the power of this approach. This has been one of Aris's main themes throughout his career, and it is a mystery to me that so many models are still reported in the literature with the original dimensions of the variables and parameters intact.

He soon gets down to the important distinctions between conservation laws and constitutive laws, and lumped and distributed parameter systems (a distinction which he admits is, "enough to frighten children in their beds.") He derives the general balance equations for distributed systems, discusses the importance, and sometimes the great difficulty, of finding the right boundary conditions. He describes these topics in the context of the tubular reactor together with the "notorious pair" of Danckwerts boundary conditions about which, "much ink has been spilt."

In Chapters 2, 3 and 4, Aris develops his themes (such as studying limiting cases, getting the most from calculations, and so on) on a wide variety of models. His frankness and his modesty is nowhere better displayed than on pages 67 et. seq. when he discusses "observing conditions." Aris is reminiscent here of P.V. Danckwerts (1982) who wrote, "I was amused to find that the treatment of the tubular reactor with longitudinal dispersion, together with the 'Danckwerts boundary conditions,' which I put forward in 1953 and which were credited to me for at least 25 years, had been anticipated by Langmuir in 1908 (I should have been less amused in 1954)."

Chapters 5 and 6 are two of the shortest and most beautiful chapters in the book. Aris's approach to modeling is succinctly summarized with his own "top 10" list of maxims in Chapter 5. These include: "Cast the problem in dimensionless form; Use limiting cases, which may be much more easily soluble, to box the problem; Make haste slowly," and my personal favorite, "Think geometrically." Chapter 6, "Style," discusses important aspects rarely touched in scientific books, including literary style, genre, plagiarism and attribution, and publish or perish. In this chapter we get not only excellent advice, but a deeper sense of Aris's values: modesty, leadership, and strongly held opinions on a wide variety of subjects. In his own words, these two chapters allow him to "get a few things off my chest."

Aris has written papers on a bewildering array of subjects; many are classics. The 17 papers reproduced in Part II represent about 5% of his bibliography, so do not be surprised if some of your personal favorites are missing. The selected papers provide canonical examples of the art and science of mathematical modeling that was laid out in Part I. Subject matter includes: dispersion in flow, formal kinetics, statics and dynamics of chemical reactors, mass and heat transfer, and modeling in general. Several major themes of Aris's bibliography are absent, including optimization (there is only passing mention of this subject on pp. 69–73, but only in the context of warning the reader to respect conditions which are easily overlooked or hidden), countercurrent devices, and so on. I especially enjoyed re-reading his papers on dispersion in flow and formal kinetics (it's a pity that his famous paper co-athored with Dick Mah (1963) was missing; this is surely one of the most important applications of linear algebra in all of chemical engineering).

The material in Part II amply demonstrates Aris's hallmark ability to write papers that leave you feeling the simplicity of the problem formulation and solution is so obvious that you too could have done it, when in fact you could not. I regularly tell my doctoral students that anybody can take a difficult problem and make it seem complicated; the trick is to make it look simple. I learned this from Aris, as did so many others.

The autobiography and bibliography in Part III makes for an interesting read and achieves the author's stated goals of providing a context, and of acknowledging the people who influenced his career.

You will find this book full of interesting gems and idiosyncrasies, such as paper H, part of which is written in the first person singular ("I submit that..." p. 216) yet the paper has two authors; the other being Neal Amundson. Perhaps, this most famous of duos has finally merged into one person. When all is said and done, the best advice I can offer is, read this book.

Literature cited

Aris, R., and R. S. Mah, "Independence of Chemical Reactions," *Ind. Eng. Chem. Fund.*, **2**, 90 (1963).

Danckwerts, P. V., "Review of *A Century of Chemical Engineering*," The Chemical Engineer, November issue, p. 406 (1982).

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